$$\therefore p=1-\frac{4}{5\pi r^3}\left[\int_{\frac{1}{2}r}^{r}\int_0^{\theta}[15(\tfrac{1}{2}\pi-\theta)\cos\theta-10\cos\theta+10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-\tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta]\times\right.$$

$$\left.x^2dxd\theta+\int_0^{\frac{1}{2}r}\int_0^{\frac{1}{2}\pi}[15(\tfrac{1}{2}\pi-\theta)\cos\theta-10\cos\theta+10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-\tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta]x^2dxd\theta\right]$$

$$=1-\frac{4}{5\pi r^3}\left[\int_{\frac{1}{2}r}^{r}\left(15(r-x)\cos^{-1}(\frac{r-x}{x})-10r-20\sqrt{(2rx-r^2)}\right.\right.$$

$$\left.\left.+\tfrac{65}{2}x-\frac{5x^2}{x+\sqrt{(2rx-r^2)}}\right)xdx+\tfrac{15}{2}\int_0^{\frac{1}{2}r}x^2dx\right]=\frac{4}{\pi}(8\log2-5).$$

Also solved with same result by the *PROPOSER*.

MISCELLANEOUS.

107. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

The index of refraction of a medium varying inversely as the square root of the distance, prove that the path of a ray of light in the medium is a cycloid.

Solution by the PROPOSER.

Taking the axis of y in the given plane and that of x at right angles to the y axis, letting $\mu=k/\sqrt{x}$ be the index of refraction, and $p=dy/dx$, we have, by the usual theory, for the differential equation to the path

$$\frac{dp/dx}{1+p^2}=\frac{1}{\mu}\left[\frac{d\mu}{dy}-\frac{d\mu}{dx}\frac{dy}{dx}\right]\text{....(1)}.\quad\text{We have }\frac{d\mu}{dx}=-\frac{k}{2x^{\frac{3}{2}}}\text{....(2)},$$

and (1) becomes $\dfrac{dp/dx}{1+p^2}=\dfrac{p}{2x}$, or $\dfrac{dp}{p(1+p^2)}=\dfrac{dx}{2x}$....(3).

Integrating, $\log\dfrac{p}{\sqrt{(1+p^2)}}=\log\sqrt{x}+C$....(4).

Let $p=b$, when $x=a$; then $C=\log\dfrac{b}{a\sqrt{(1+b^2)}}$,

and (4) becomes $p=\dfrac{dy}{dx}=\dfrac{xdx}{\sqrt{[(a^2/b^2)(1+b^2)x-x^2]}}$....(5), the differential equation to a cycloid.

Also solved by *G. B. M. ZERR*, and *L. C. WALKER*.

108. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

To divide the arc of a cardioid into eight equal parts.